OPTIMIZATION OF FUZZY INVENTORY MODEL WITH TRAPEZOIDAL FUZZY NUMBERS: LAGRANGEAN METHOD

K.Kalaiarasi Assistant Professor, Department of Mathematics, Cauvery College for women (Autonomous), (Affiliated to Bharathidasan University), Tiruchirapalli, Tamilnadu-620018, India
Email id: kalaishruthi12@gmail.com
R.Abarna PG student, Department of Mathematics, Cauvery College for women (Autonomous), (Affiliated to Bharathidasan University), Tiruchirapalli, Tamilnadu-620018, India
Email id: abibhuvanesh99@gmail.com

Abstract
In view of this paper we talk about optimal ordering strategy in inventory model with degenerate materials in both crisp and fuzzy model. Here $h, \widetilde{w},$ are taken as trapezoidal fuzzy numbers and defuzzify the model by using graded mean integration .The fundamental target of this paper is to decide the optimal production quantity and minimize the total cost for the proper total cost. The ideal strategy of the fuzzy creation stock model and fuzzy complete expense is assessed by utilizing the calculation augmentation of a lagrangean technique. A mathematical model is utilized to show the best correlation between the combination fuzzy models.

Keywords Total cost, Lagrange method, Graded mean integration, Trapezoidal fuzzy number.

1. Introduction

In 2007, the concept of Minimizing the Economic lot size of a three-stage supply chain are introduced by C.J.Chung, H.M.Wee. In 1990, the concept of Economic ordering policies during special discount periods for dynamic inventory problems are developed by S.K.Goyal. In Kalaiarasi K., Sumathi M .,Sabina begum M., [9] analyzed Optimization of fuzzy inventory model for Economic Order Quantity using Lagragian method. In Kalaiarasi K., Sumathi M., And Daisy S., [10] developed the Fuzzy Economic Order Quantity Inventory Model Using Lagrangian method.

In Section 2, represents graded mean integration and some arithmetic operations. In Section 3, inventory for crisp model and fuzzy model are presented. Numerical example is given to test the proposed model and Sensitivity analysis has been made for different changes in the parameter values in section 4, finally conclusion have been made in section 5.

2. Methodology

2.1 Graded Mean Integration For Trapezoidal Numbers

$$P(\tilde{Y}) = \frac{\int_0^1 \frac{h}{2}\left[y_1 + y_4 + hy_2 - y_1 - y_4 + y_3\right]dh}{\int_0^1 hdh} = \frac{y_1 + 2y_2 + 2y_3 + y_4}{6}$$

2.2 Fuzzy Arithmetical Operations

Suppose we take two trapezoidal fuzzy numbers namely $\tilde{X} = (u_1, u_2, u_3, u_4)$ & $\tilde{Y} = (v_1, v_2, v_3, v_4)$ respectively. Then

(1) $\tilde{X} \oplus \tilde{Y} = (u_1 + v_1, u_2 + v_2, u_3 + v_3, u_4 + v_4)$

(2) $\tilde{X} \otimes \tilde{Y} = (u_1 v_1, u_2 v_2, u_3 v_3, u_4 v_4)$

(3) $-\tilde{Y} = (-v_4, -v_3, -v_2, -v_1)$, $\tilde{X} - \tilde{Y} = (u_1 - v_4, u_2 - v_3, u_3 - v_2, u_4 - v_1)$

(4) $\frac{1}{\tilde{Y}} = \tilde{Y}^{-1} = \left(\frac{1}{v_4}, \frac{1}{v_3}, \frac{1}{v_2}, \frac{1}{v_1}\right)$, $\frac{\tilde{X}}{\tilde{Y}} = \left(\frac{u_1}{v_4}, \frac{u_2}{v_3}, \frac{u_3}{v_2}, \frac{u_4}{v_1}\right)$

(5) Let $\alpha \in R$, then

(i) $\alpha \geq 0, \alpha \otimes \tilde{X} = (\alpha u_1, \alpha u_2, \alpha u_3, \alpha u_4)$

(ii) $\alpha \leq 0, \alpha \otimes X^{\sim} = (\alpha u_4, \alpha u_3, \alpha u_2, \alpha u_1)$

3. Inventory Model Development

Consider the total cost,

$$Z(h,w) = \left[\frac{s_n - s_m}{R} - \frac{n_y(h-1)u}{2R}\right]\left[\frac{a^{\mu R}\left(w\mu^2 + b\mu + bc - bcR\mu\right) - b(\mu + c) - w\mu^2}{\mu^2}\right]$$

$$-\frac{I}{R} - \frac{d_0}{hR} - \frac{n_c}{R}\left(\frac{-w}{\mu} + b\left(\frac{c\mu^2R^2 - 2\mu - 2c - 2\mu^2R^2}{2\mu^3}\right)\right) + \frac{n_c a^{\mu R}}{R}\left(b\left(\frac{\mu cR - \mu - c}{\mu^3}\right) - \frac{w}{\mu}\right)$$

Now we differentiate w.r.t h and equate it to zero, then we obtain the crisp production quantity

$$h = \sqrt{\frac{2d_0\mu^2}{n_y u.a^{\mu R}(w\mu^2 + b\mu + bc - bcR\mu) - b(\mu + c) - w\mu^2}}$$

3.1 Inventory for Crisp Model

Over this paper, we utilization of the accompanying factors to improve the treatment of a coordinated stock model. Here h, $\widetilde{w}$ are fuzzy parameters. The annual total cost is,

$$J\tilde{T}c(h,w) = \begin{bmatrix} \left(\left(\frac{s_n - s_m}{R} - \frac{n_y(h-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_1\mu^2 + b\mu - bcR\mu) - b(\mu + c) - w_1\mu^2}{\mu^2}\right) - \frac{I}{R} - \frac{d_0}{hR} \right. \\ \left. -\frac{n_c}{R}\left(\frac{-w_1}{\mu} + b\left(\frac{c\mu^2R^2 - 2\mu - 2c - 2\mu^2R^2}{2\mu^3}\right)\right) + \frac{n_c a^{\mu R}}{R} b\left(\frac{\mu cR - \mu - c}{\mu^3}\right) - \frac{w_1}{\mu}\right), \\ \left(\left(\frac{s_n - s_m}{R} - \frac{n_y(h-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_2\mu^2 + b\mu - bcR\mu) - b(\mu + c) - w_2\mu^2}{\mu^2}\right) - \frac{I}{R} - \frac{d_0}{hR} \right. \\ \left. -\frac{n_c}{R}\left(\frac{-w_2}{\mu} + b\left(\frac{c\mu^2R^2 - 2\mu - 2c - 2\mu^2R^2}{2\mu^3}\right)\right) + \frac{n_c a^{\mu R}}{R} b\left(\frac{\mu cR - \mu - c}{\mu^3}\right) - \frac{w_2}{\mu}\right), \\ \left(\left(\frac{s_n - s_m}{R} - \frac{n_y(h-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_3\mu^2 + b\mu - bcR\mu) - b(\mu + c) - w_3\mu^2}{\mu^2}\right) - \frac{I}{R} - \frac{d_0}{hR} \right. \\ \left. -\frac{n_c}{R}\left(\frac{-w_3}{\mu} + b\left(\frac{c\mu^2R^2 - 2\mu - 2c - 2\mu^2R^2}{2\mu^3}\right)\right) + \frac{n_c a^{\mu R}}{R} b\left(\frac{\mu cR - \mu - c}{\mu^3}\right) - \frac{w_3}{\mu}\right), \\ \left(\left(\frac{s_n - s_m}{R} - \frac{n_y(h-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_4\mu^2 + b\mu - bcR\mu) - b(\mu + c) - w_4\mu^2}{\mu^2}\right) - \frac{I}{R} - \frac{d_0}{hR} \right. \\ \left. -\frac{n_c}{R}\left(\frac{-w_4}{\mu} + b\left(\frac{c\mu^2R^2 - 2\mu - 2c - 2\mu^2R^2}{2\mu^3}\right)\right) + \frac{n_c a^{\mu R}}{R} b\left(\frac{\mu cR - \mu - c}{\mu^3}\right) - \frac{w_4}{\mu}\right) \end{bmatrix}$$

Now, we Defuzzify the fuzzy total cost by using Graded Mean Integration formula, we get

$$P\left(J\tilde{T}c(h,w)\right)=\frac{1}{6}\left[\begin{array}{l}\left(\left(\frac{s_n-s_m}{R}-\frac{n_y(h-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_1\mu^2+b\mu-bcR\mu)-b(\mu+c)-w_1\mu^2}{\mu^2}\right)-\frac{I}{R}-\frac{d_0}{hR}\right. \\ \left.-\frac{n_c}{R}\left(\frac{-w_1}{\mu}+b\left(\frac{c\mu^2R^2-2\mu-2c-2\mu^2R^2}{2\mu^3}\right)\right)+\frac{n_c a^{\mu R}}{R}b\left(\frac{\mu cR-\mu-c}{\mu^3}\right)-\frac{w_1}{\mu}\right)+ \\ \left(\left(\frac{s_n-s_m}{R}-\frac{n_y(h-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_2\mu^2+b\mu-bcR\mu)-b(\mu+c)-w_2\mu^2}{\mu^2}\right)-\frac{I}{R}-\frac{d_0}{hR}\right. \\ \left.-\frac{n_c}{R}\left(\frac{-w_2}{\mu}+b\left(\frac{c\mu^2R^2-2\mu-2c-2\mu^2R^2}{2\mu^3}\right)\right)+\frac{n_c a^{\mu R}}{R}b\left(\frac{\mu cR-\mu-c}{\mu^3}\right)-\frac{w_2}{\mu}\right)+ \\ \left(\left(\frac{s_n-s_m}{R}-\frac{n_y(h-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_3\mu^2+b\mu-bcR\mu)-b(\mu+c)-w_3\mu^2}{\mu^2}\right)-\frac{I}{R}-\frac{d_0}{hR}\right. \\ \left.-\frac{n_c}{R}\left(\frac{-w_3}{\mu}+b\left(\frac{c\mu^2R^2-2\mu-2c-2\mu^2R^2}{2\mu^3}\right)\right)+\frac{n_c a^{\mu R}}{R}b\left(\frac{\mu cR-\mu-c}{\mu^3}\right)-\frac{w_3}{\mu}\right)+ \\ \left(\left(\frac{s_n-s_m}{R}-\frac{n_y(h-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_4\mu^2+b\mu-bcR\mu)-b(\mu+c)-w_4\mu^2}{\mu^2}\right)-\frac{I}{R}-\frac{d_0}{hR}\right. \\ \left.-\frac{n_c}{R}\left(\frac{-w_4}{\mu}+b\left(\frac{c\mu^2R^2-2\mu-2c-2\mu^2R^2}{2\mu^3}\right)\right)+\frac{n_c a^{\mu R}}{R}b\left(\frac{\mu cR-\mu-c}{\mu^3}\right)-\frac{w_4}{\mu}\right)\end{array}\right]$$

To find the minimization of P(JTC(h,w)), first we differentate partially w.r.t h and equate it to zero, we get,

$\frac{\partial P(JT^{\sim}C(h,w))}{\partial h}=0$ .Then we get,

$$h=\sqrt{\frac{8d_0\mu^2}{n_y u\left[a^{\mu R}((w_1+w_2+w_3+w_4)\mu^2+4b\mu+4bc-4bcR\mu)-4b(\mu+c)-(w_1+w_2+w_3+w_4)\mu^2\right]}}$$

3.2 Inventory for fuzzy Model

Let $\tilde{h}$ be a trapezoidal fuzzy number $h^{\sim}=(h_1,h_2,h_3,h_4)$ with $0<h_1\le h_2\le h_3\le h_4$ and defuzzify the Total cost by using graded mean defuzzification.

Therefore the total cost for this model is,

$$J\tilde{T}c(h,w)=\frac{1}{6}\left[\begin{array}{l}\left(\begin{array}{l}\left(\frac{s_n-s_m}{R}-\frac{n_y(h_1-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_1\mu^2+b\mu-bcR\mu)-b(\mu+c)-w_1\mu^2}{\mu^2}\right)-\frac{I}{R}-\frac{d_0}{h_4R}\\ -\frac{n_c}{R}\left(\frac{-w_1}{\mu}+b\left(\frac{c\mu^2R^2-2\mu-2c-2\mu^2R^2}{2\mu^3}\right)\right)+\frac{n_ca^{\mu R}}{R}b\left(\frac{\mu cR-\mu-c}{\mu^3}\right)-\frac{w_1}{\mu}\end{array}\right)+\\ \left(\begin{array}{l}\left(\frac{s_n-s_m}{R}-\frac{n_y(h_2-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_2\mu^2+b\mu-bcR\mu)-b(\mu+c)-w_2\mu^2}{\mu^2}\right)-\frac{I}{R}-\frac{d_0}{h_3R}\\ -\frac{n_c}{R}\left(\frac{-w_2}{\mu}+b\left(\frac{c\mu^2R^2-2\mu-2c-2\mu^2R^2}{2\mu^3}\right)\right)+\frac{n_ca^{\mu R}}{R}b\left(\frac{\mu cR-\mu-c}{\mu^3}\right)-\frac{w_2}{\mu}\end{array}\right)+\\ \left(\begin{array}{l}\left(\frac{s_n-s_m}{R}-\frac{n_y(h_3-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_3\mu^2+b\mu-bcR\mu)-b(\mu+c)-w_3\mu^2}{\mu^2}\right)-\frac{I}{R}-\frac{d_0}{h_2R}\\ -\frac{n_c}{R}\left(\frac{-w_3}{\mu}+b\left(\frac{c\mu^2R^2-2\mu-2c-2\mu^2R^2}{2\mu^3}\right)\right)+\frac{n_ca^{\mu R}}{R}b\left(\frac{\mu cR-\mu-c}{\mu^3}\right)-\frac{w_3}{\mu}\end{array}\right)+\\ \left(\begin{array}{l}\left(\frac{s_n-s_m}{R}-\frac{n_y(h_4-1)u}{2R}\right)\left(\frac{a^{\mu R}(w_4\mu^2+b\mu-bcR\mu)-b(\mu+c)-w_4\mu^2}{\mu^2}\right)-\frac{I}{R}-\frac{d_0}{h_1R}\\ -\frac{n_c}{R}\left(\frac{-w_4}{\mu}+b\left(\frac{c\mu^2R^2-2\mu-2c-2\mu^2R^2}{2\mu^3}\right)\right)+\frac{n_ca^{\mu R}}{R}b\left(\frac{\mu cR-\mu-c}{\mu^3}\right)-\frac{w_4}{\mu}\end{array}\right)\end{array}\right],$$

With $0<h_1\le h_2\le h_3\le h_4$ into the following inequality $h_2-h_1\ge 0, h_3-h_2\ge 0, h_4-h_3\ge 0, h_1>0$. In the accompanying advances augmentation of the Lagrangean strategy is utilized to discover the arrangements of h_1,h_2,h_3,h_4 to minimize $P(JT^{\sim}C(h,w))$ in above formula.

STEP 1:

Work out the unconstraint problem. To determine the min $P(JT^{\sim}C_1(h,w))$ we have to find the derivative of $P(JT^{\sim}C_1(h,w))$ with respect to h_1,h_2,h_3,h_4 and equate it to zero. We get,

Let $\frac{\partial P}{\partial h_1}=0$ implies $h_1=\sqrt{\frac{2d_0\mu^2}{n_yu\left[a^{\mu R}(w_1\mu^2+b\mu+bc-bcR\mu)-b(\mu+c)-w_1\mu^2\right]}}$

Let $\frac{\partial P}{\partial h_2}=0$ implies $h_2=\sqrt{\frac{2d_0\mu^2}{n_yu\left[a^{\mu R}(w_2\mu^2+b\mu+bc-bcR\mu)-b(\mu+c)-w_2\mu^2\right]}}$

Let $\frac{\partial P}{\partial h_3}=0$ implies $h_3=\sqrt{\frac{2d_0\mu^2}{n_yu\left[a^{\mu R}(w_3\mu^2+b\mu+bc-bcR\mu)-b(\mu+c)-w_3\mu^2\right]}}$

Let $\frac{\partial P}{\partial h_4}=0$ implies $h_4=\sqrt{\frac{2d_0\mu^2}{n_yu\left[a^{\mu R}(w_4\mu^2+b\mu+bc-bcR\mu)-b(\mu+c)-w_4\mu^2\right]}}$

Because the above values $h_1>h_2>h_3>h_4$.

It does not satisfy the constraint $0<h_1>h_2>h_3>h_4$. Then set S=1 and move to step 2.

STEP 2: The inequality constraint $h_2-h_1\ge 0$ transform into equality constraint $h_2-h_1=0$. Then the Lagrangean method is,

$$L(h_1,h_2,h_3,h_4,\lambda)=P[JT^{\sim}C_1(h,w)]-\lambda(h_2-h_1)$$

To find the minimization of $L(h_1,h_2,h_3,h_4,\lambda)$,

$$h_1 = h_2 = \sqrt{\frac{4d_0\mu^2}{n_y u\left[a^{\mu R}((w_1+w_2)\mu^2 + 2b\mu + 2bc - 2bcR\mu) - 2b(\mu + c) - (w_1+w_2)\mu^2\right]}}$$

$$h_3 = \sqrt{\frac{2d_0\mu^2}{n_y u\left[a^{\mu R}(w_3\mu^2 + b\mu + bc - bcR\mu) - b(\mu + c) - w_3\mu^2\right]}}$$

$$h_4 = \sqrt{\frac{2d_0\mu^2}{n_y u\left[a^{\mu R}(w_4\mu^2 + b\mu + bc - bcR\mu) - b(\mu + c) - w_4\mu^2\right]}}$$

Because the above value. It does not satisfy the constraints. The set S=2 and move to step 3.

STEP 3:

The inequality constraints $h_2 - h_1 \geq 0$, transform into equality constraints $h_2 - h_1 = 0$ and $h_3 - h_1 = 0$. We optimize $P(JT^{\sim}C_1(h,w))$Then the Lagrangean method is,
$L(h_1, h_2, h_3, h_4, \lambda_1, \lambda_2) = P[JT^{\sim}C_1(h,w) - \square_1(h_2 - h_1) - \square_2(h_3 - h_2)$.
To find the minimization of $L(h_1, h_2, h_3, h_4, \lambda_1, \lambda_2)$,

$$h_1 = h_2 = h_3 = \sqrt{\frac{6d_0\mu^2}{n_y u\left[a^{\mu R}((w_1+w_2+w_3)\mu^2 + 3b\mu + 3bc - 3bcR\mu) - 3b(\mu + c) - (w_1+w_2+w_3)\mu^2\right]}}$$

$$h_4 = \sqrt{\frac{2d_0\mu^2}{n_y u\left[a^{\mu R}(w_4\mu^2 + b\mu + bc - bcR\mu) - b(\mu + c) - w_4\mu^2\right]}}$$

Because the above value. It does not satisfy the constraints. The set S=3 and go to step 4.

STEP 4:

The inequality constraints transform into equality constraints , Then the lagrangean method is ,
$L(h_1, h_2, h_3, h_4, \lambda_1, \lambda_2, \lambda_3) = P[JT^{\sim}C_1(h, \square) - \square_1(h_2 - h_1) - \square_2(h_3 - h_2) - \square_3(h_4 - h_3)$.

$$h_1 = h_2 = h_3 = h_4 = \sqrt{\frac{8d_0\mu^2}{n_y u\left[a^{\mu R}((w_1+w_2+w_3+w_4)\mu^2 + 4b\mu + 4bc - 4bcR\mu) - 4b(\mu + c) - (w_1+w_2+w_3+w_4)\mu^2\right]}}$$

The above results satisfies all inequality constraints. Let $h_1 = h_2 = h_3 = h_4 = \tilde{h}^*$.
Then the optimal production quantity value is,

$$\tilde{h}^* = \sqrt{\frac{8d_0\mu^2}{n_y u\left[a^{\mu R}((w_1+w_2+w_3+w_4)\mu^2 + 4b\mu + 4bc - 4bcR\mu) - 4b(\mu + c) - (w_1+w_2+w_3+w_4)\mu^2\right]}}$$

4. Numerical Example

we Consider the characteristics as follows.
$\mu = 0.9, d_0 = 85, n_y = 0.3, u = 1, a = 100, R = 2, b = 0.1, c = 0.4, s_n = 2.5, s_m = 0.8, I = 9, n_c = 0.5, w = 0.6$

Crisp model

h =0.460435355,JTc(h,w) =1540.941445

Fuzzy model

$\tilde{h}^* =$ 0.460435355 , $J\tilde{T}c(h,w) =$ 1540.941445.

Graphical representation of the above results is given by,

 ISSN: 0975-4520
Vol-23 No.04(IX) October-December 2020

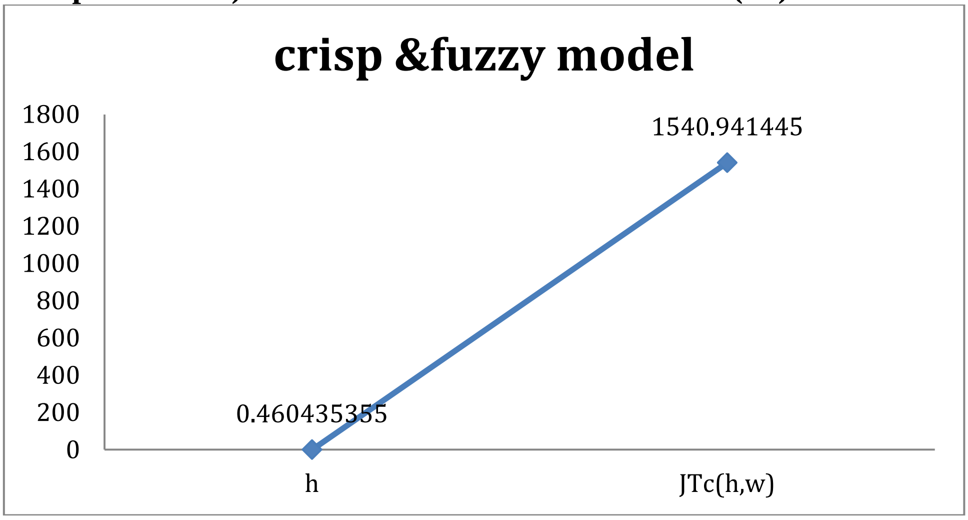

Fig 4.1

If d_o value increased, then the results of order quantity and total cost are given as follows.

d_o	H	JTc(h,w)
85	0.460435355	1540.941445
90	0.473784073	1540.866048
95	0.486766862	1540.655895
100	0.499412263	1540.331618
105	0.511745287	1539.910100

Graphical representation of the above table is given by,

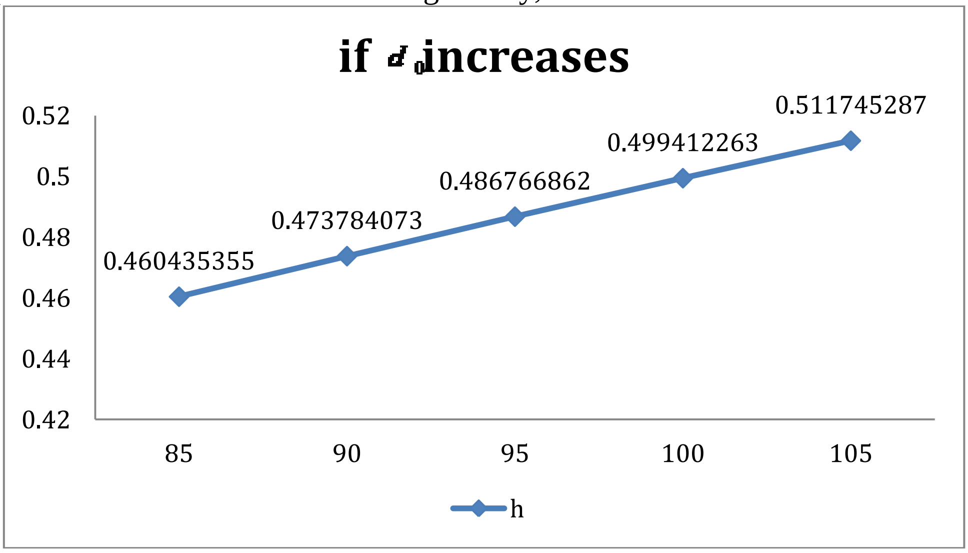

Fig 4.2

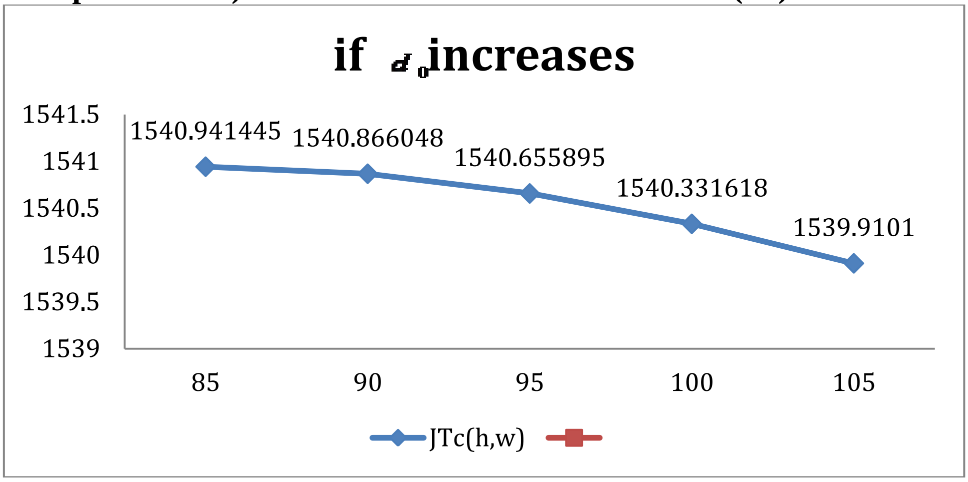

Fig 4.3

5. CONCLUSION

In view of this paper, we have discussed order quantity and total cost in both crisp and fuzzy models. The parameters are taken as trapezoidal fuzzy numbers and defuzzified the total cost by graded mean integration method. This model has been solved by lagrangean method. We conclude that fuzzy economic production quantity obtained is very closed to crisp economic production quantity. Finally, numerical example, sensivity analysis and some graphs are given to the proposed model.

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