EXPECTED PROFIT OF AN EOQ INVENTORY MODEL WITH INFERIOR WORTH PRODUCTS

K.Kalaiarasi, **R.Sowmiya
Assistant professor, Department of Mathematics, Cauvery College for women (Autonomous), (Affiliated to Bharathidasan University), Tiruchirapalli, Mail id: kalaishruthi12@gmail.com
PG student, Department of Mathematics, Cauvery College for women (Autonomous), (Affiliated to Bharathidasan University), :: rajasathamangalam20@gmail.com

Abstract

In supply chain management, reorder point is significant for an inventory system.In this paper we compute maximizing order quality and minimizing total cost utilizing lagrange method. The inventory parameters are converted to fuzzy parameters using fuzzy trapezoidal numbers. Graded mean integration method is used for defuzzification. In this way, a numerical example is settled to acquire for used to see the integration models.

Keywords: EOQ inventory, Total cost, Fuzzification, Defuzzification, Graded Mean Integration, Trapezoidal number.

1. INTRODUCTION

The supply chain network design (SCND) choice has critical effect on the exhibition of the supply chain (SC) because it influences complete inventory and transportation costs in a since quite a while ago run[1] .Short-lived items are often found in business and industry. The basic criticalness of transient items in numerous modern areas, from food, drug and medical care to high-innovation businesses, is especially evident. A fuzzy methodology is more qualified to a help process than a crisp one.

The Economic Order Quantity (EOQ) model was created by Ford W.Harris in 1913[2,3].The idea of fuzzy sets was presented by Lofti A.Zadeh (1965)[4]. L.A Zadeh and R.E.Bellman [5] were presented fuzzy set hypothesis in decision making process. K.Kalaiarasi,M.Sumathi, [6-9] analyzed EOQ model optimized using non linear programming methods .

In this paper total cost was derived from Nobil [10].optimized the total cost using Lagragean method,the inventory paremeters are fuzzified by trapezoidal fuzzy numbers. GM integration is used for defuzzifed. The numerical example of the mathematical model represents the arrangement methodology exhibiting that the created model.

2.METHODOLOGY

2.1. GRADED MEAN INTEGRATION REPRESENTATION METHOD

The Graded mean integration representation of $\tilde{Z}$ as

$$P(\tilde{Z})=\frac{\int_0^1 \frac{H}{2}[(z_1+z_4)+H(z_2-z_1-z_4+z_3)]dH}{\int_0^1 H\,dH}=\frac{z_1+2z_2+2z_3+z_4}{6} \qquad \ldots(s)$$

3. MATHEMATICAL MODEL

3.1. NOTATIONS

M ⟶ Optimum batch size (Decision variables)
R ⟶ Demand rate
L ⟶ Fixed ordering cost
H ⟶ Holding cost per item

The Derived the total profit is

$$TP(M)=\frac{\square R-\frac{RL}{M}-HM\gamma}{\delta}$$

Differentiating with respect to M and solving M is

$$M^*=\sqrt{\frac{\delta(RL)}{H\gamma}}$$

4.An integrated inventory models:

4.1 crisp case

The annual integrated total inventory cost for expected profit

$$J\tilde{T}C(M) = \left\{ \left[\frac{\beta R_1 - \frac{R_1L_1}{M} - H_1M\gamma}{\delta}\right], \left[\frac{\beta R_2 - \frac{R_2L_2}{M} - H_2M\gamma}{\delta}\right], \left[\frac{\beta R_3 - \frac{R_3L_3}{M} - H_3M\gamma}{\delta}\right], \left[\frac{\beta R_4 - \frac{R_4L_4}{M} - H_4M\gamma}{\delta}\right] \right\}$$

$$P(J\tilde{T}C(M)) = \frac{1}{6}\left\{ \left[\frac{\beta R_1 - \frac{R_1L_1}{M} - H_1M\gamma}{\delta}\right] + 2\left[\frac{\beta R_2 - \frac{R_2L_2}{M} - H_2M\gamma}{\delta}\right] + 2\left[\frac{\beta R_3 - \frac{R_3L_3}{M} - H_3M\gamma}{\delta}\right] + \left[\frac{\beta R_4 - \frac{R_4L_4}{M} - H_4M\gamma}{\delta}\right] \right\}$$

To find the minimization of $P(J\tilde{T}C(M))$,

$$\frac{\partial P(J\tilde{T}C(M))}{\partial M} = \frac{1}{6}\left\{ \left[\frac{\delta\left(\frac{R_1L_1}{M^2} - H_1\gamma\right)}{\delta^2}\right] + 2\left[\frac{\delta\left(\frac{R_2L_2}{M^2} - H_2\gamma\right)}{\delta^2}\right] + 2\left[\frac{\delta\left(\frac{R_3L_3}{M^2} - H_3\gamma\right)}{\delta^2}\right] + \left[\frac{\delta\left(\frac{R_4L_4}{M^2} - H_4\gamma\right)}{\delta^2}\right] \right\}$$

Let $\frac{\partial P(J\tilde{T}C(M))}{\partial M} = 0$. ***we get,***

$$M^* = \sqrt{\frac{R_1L_1 + 2R_2L_2 + 2R_3L_3 + R_4L_4}{H_1\gamma + 2H_2\gamma + 2H_3\gamma + H_4\gamma}}$$

4.2. Fuzzy Case

we take the fuzzy total production inventory cost

$$p(J\tilde{T}C_1(M)) = \frac{1}{6}\left\{ \left[\frac{\beta R_1 - \frac{R_1L_1}{M} - H_1M_1\gamma}{\delta}\right], 2\left[\frac{\beta R_2 - \frac{R_2L_2}{M} - H_2M_2\gamma}{\delta}\right], 2\left[\frac{\beta R_3 - \frac{R_3L_3}{M} - H_3M_3\gamma}{\delta}\right], \left[\frac{\beta R_4 - \frac{R_4L_4}{M} - H_4M_4\gamma}{\delta}\right] \right\}$$

we apply the graded mean integration representation of $p(J\tilde{T}C_1(M))$ is

$$p(J\tilde{T}C_1(M)) = \frac{1}{6}\left\{ \left[\frac{\beta R_1 - \frac{R_1L_1}{M_4} - H_1M_1\gamma}{\delta}\right] + 2\left[\frac{\beta R_2 - \frac{R_2L_2}{M_3} - H_2M_2\gamma}{\delta}\right] + 2\left[\frac{\beta R_3 - \frac{R_3L_3}{M_2} - H_3M_3\gamma}{\delta}\right] + \right.$$

βR4−R4L4M1−H4M4γδ …(c)

with $0 < M_1 \le M_2 \le M_3 \le M_4$ .we transforms this inequality $M_1 - M_2 \ge 0$, $M_3 - M_2 \ge 0$, $M_4 - M_3 \ge 0$, $M_1 > 0$.Lagrangean method is used to find M_1, M_2, M_3, M_4 .

Case 1: To find the min $P(J\tilde{T}C(M))$,

$$\frac{\partial P}{\partial M_1} = \frac{1}{6}\left\{\frac{\delta[-H_1\gamma]}{\delta^2} + \frac{\frac{\delta[R_4L_4]}{M_1^2}}{\delta^2}\right\}, \frac{\partial P}{\partial M_2} = \frac{1}{6}\left\{2\frac{\delta[-H_2\gamma]}{\delta^2} + 2\frac{\frac{\delta[R_3L_3]}{M_2^2}}{\delta^2}\right\}$$

$$\frac{\partial P}{\partial M_3} = \frac{1}{6}\left\{2\frac{\delta[-H_3\gamma]}{\delta^2} + 2\frac{\frac{\delta[R_2L_2]}{M_3^2}}{\delta^2}\right\}, \frac{\partial P}{\partial M_4} = \frac{1}{6}\left\{\frac{\delta[-H_4\gamma]}{\delta^2} + \frac{\frac{\delta[R_1L_1]}{M_4^2}}{\delta^2}\right\}$$

$$\frac{\partial P}{\partial M_1} = 0 . \Rightarrow M_1 = \sqrt{\frac{R_4L_4}{H_1\gamma}}, \frac{\partial P}{\partial M_2} = 0. \Rightarrow M2 = \sqrt{\frac{2R_3L_3}{2H_2\gamma}}$$

$$\frac{\partial P}{\partial M_3} = 0. \Rightarrow M_3 = \sqrt{\frac{2R_2L_2}{2H_3\gamma}}, \frac{\partial P}{\partial M_4} = 0. \Rightarrow M4 = \sqrt{\frac{R_1L_1}{H_4\gamma}}$$

$relation\ is\ not\ satisfied\ \ 0 < M_1 > M_2 > M_3 > M_4$.**So put N=1 and go to case 2.**

Case 2: Convert the inequality constraint $M_2 - M_1 \geq 0$ into equality constraint $M_2 - M_1 = 0$ We have Lagragean method as

L_r**(M1,M2,M3,M4,** λ**)= P[J**$\tilde{T}C_1(M)$]-λ**(**$M_2 - M_1$)

Then

$$\frac{\partial L_r}{\partial M_1} = \frac{1}{6}\left\{\frac{\delta[-H_1\gamma]}{\delta^2} + \frac{\frac{\delta[R_4L_4]}{M_1^2}}{\delta^2}\right\} + \lambda = 0, \frac{\partial L_r}{\partial M_2} = \frac{1}{6}\left\{2\frac{\delta[-H_2\gamma]}{\delta^2} + 2\frac{\frac{\delta[R_3L_3]}{M_2^2}}{\delta^2}\right\} - \lambda = 0$$

$$\frac{\partial L_r}{\partial M_3} = \frac{1}{6}\left\{2\frac{\delta[-H_3\gamma]}{\delta^2} + 2\frac{\frac{\delta[R_2L_2]}{M_3^2}}{\delta^2}\right\} = 0, \frac{\partial L_r}{\partial M_4} = \frac{1}{6}\left\{\frac{\delta[-H_4\gamma]}{\delta^2} + \frac{\frac{\delta[R_1L_1]}{M_4^2}}{\delta^2}\right\} = 0$$

$$\frac{\partial L_r}{\partial \lambda} = -(M_2 - M_1).$$

$$\Rightarrow M_1 = M_2 = \sqrt{\frac{R_4L_4 + 2R_3L_3}{H_1\gamma + 2H_2\gamma}}, M_3 = \sqrt{\frac{2R_2L_2}{2H_3\gamma}}, M4 = \sqrt{\frac{R_1L_1}{H_4\gamma}}$$

relation is not satisfied $0 < M_1 > M_2 > M_3 > M_4$.**So put N=2 and go to case 3.**

Case 3: Convert the inequality constraints $M_2 - M_1 \geq 0,$ *into equality constraints* $M_2 - M_1 = 0$ *snd* $M_3 - M_2 = 0$. **The lagragean method is**

$$L_r(M_1, M_2, M_3, M_4, \lambda_1, \lambda_2) = P[J\tilde{T}C_1(M) - \lambda_1(M_2 - M_1) - \lambda_2(M_3 - M_2)$$

$$\frac{\partial L_r}{\partial M_1} = \frac{1}{6}\left\{\frac{\delta[-H_1\gamma]}{\delta^2} + \frac{\frac{\delta[R_4L_4]}{M_1^2}}{\delta^2}\right\} + \lambda_1 = 0, \frac{\partial L_r}{\partial M_2} = \frac{1}{6}\left\{2\frac{\delta[-H_2\gamma]}{\delta^2} + 2\frac{\frac{\delta[R_3L_3]}{M_2^2}}{\delta^2}\right\} - \lambda_1 + \lambda_2 = 0$$

$$\frac{\partial L_r}{\partial M_3} = \frac{1}{6}\left\{2\frac{\delta[-H_3\gamma]}{\delta^2} + 2\frac{\frac{\delta[R_2L_2]}{M_3^2}}{\delta^2}\right\} - \lambda_2 = 0, \frac{\partial L_r}{\partial M_4} = \frac{1}{6}\left\{\frac{\delta[-H_4\gamma]}{\delta^2} + \frac{\frac{\delta[R_1L_1]}{M_4^2}}{\delta^2}\right\} = 0$$

Equal the partial derivatives terms equal to zero.

$$\frac{\partial L_r}{\partial M_1} = 0 \quad M_1 = \sqrt{\frac{R_4L_4 + 6\delta\lambda_1}{H_1\gamma}}, \quad \frac{\partial L_r}{\partial M_2} = 0 \quad M_2 = \sqrt{\frac{2R_3L_3 - 6\delta\lambda_1 + 6\delta\lambda_2}{2H_2\gamma}}$$

$$\frac{\partial L_r}{\partial M_3} = 0, \quad M_3 = \sqrt{\frac{2R_2L_2 - 6\delta\lambda_2}{2H_3\gamma}}, \quad \frac{\partial L_r}{\partial M_4} = 0, \quad M_4 = \sqrt{\frac{R_1L_1}{H_4\gamma}}$$

$$\frac{\partial L_r}{\partial \lambda_1} = -(M_2 - M_1), \quad \frac{\partial L_r}{\partial \lambda_2} = -(M_3 - M_2)$$

$$\Rightarrow M_1 = M_2 = M_3 = \sqrt{\frac{R_4L_4 + 2R_3L_3 + 2R_2L_2}{H_1\gamma + 2H_2\gamma + 2H_3\gamma}}, \quad M4 = \sqrt{\frac{R_1L_1}{H_4\gamma}}$$

relation is not satisfied $0 < M_1 > M_2 > M_3 > M_4$.,**So put N=3 and go to case 4.**

Case 4: Convert the inequality constraints $M_2 - M_1 \geq 0$, $M_3 - M_2 \geq 0$ **and** $M_4 - M_3 \geq 0$ **into equality constraints** $M_2 - M_1 = 0$, $M_3 - M_2 = 0$ **and** $M_4 - M_3 = 0$ **.**

Then the lagragean function is

L_r **(**$M_1, M_2, M_3, M_4, \lambda_1, \lambda_2, \lambda_3$) = **P[J**$\tilde{T}C_1(M)$]- $\lambda_1(M_2 - M_1) - \lambda_2(M_3 - M_2) - \lambda_3(M_4 - M_3)$

$$\frac{\partial L_r}{\partial M_1} = \frac{1}{6}\left\{\frac{\delta[-H_1\gamma]}{\delta^2} + \frac{\frac{\delta[R_4L_4]}{M_1^2}}{\delta^2}\right\} + \lambda_1, \frac{\partial L_r}{\partial M_2} = \frac{1}{6}\left\{2\frac{\delta[-H_2\gamma]}{\delta^2} + 2\frac{\frac{\delta[R_3L_3]}{M_2^2}}{\delta^2}\right\} - \lambda_1 + \lambda_2 = 0$$

$$\frac{\partial L_r}{\partial M_3} = \frac{1}{6}\left\{2\frac{\delta[-H_3\gamma]}{\delta^2} + 2\frac{\frac{\delta[R_2L_2]}{M_3^2}}{\delta^2}\right\} - \lambda_2 + \lambda_3 = 0, \frac{\partial L_r}{\partial M_4} = \frac{1}{6}\left\{\frac{\delta[-H_4\gamma]}{\delta^2} + \frac{\frac{\delta[R_1L_1]}{M_4^2}}{\delta^2}\right\} - \lambda_3 = 0$$

$$\frac{\partial L_r}{\partial \lambda_1} = -(M_2 - M_1), \frac{\partial L_r}{\partial \lambda_2} = -(M_3 - M_2), \frac{\partial L_r}{\partial \lambda_3} = -(M_4 - M_3)$$

$$M_1 = M_2 = M_3 = M_4 = \sqrt{\frac{R_4 L_4 + 2R_3 L_3 + 2R_2 L_2 + R_1 L_1}{H_1\gamma + 2H_2\gamma + 2H_3\gamma + H_4\gamma}}$$

Then the optimal fuzzy production quantity is

$\widetilde{M}^* =$ **(M* , M* , M* , M*)**

$$M^* = \sqrt{\frac{R_1L_1+2R_2L_2+2R_3L_3+R_4L_4}{H_1\gamma+2H_2\gamma+2H_3\gamma+H_4\gamma}}$$

5.NUMERICAL ANALYSIS

5.1.Crisp model

The crisp parameters are

R=2500, L=100,H=5, $\beta = 3, \gamma = 0.98, \delta = 10$

M* =714.2857142857 and JTC(M)=365

5.2.Fuzzy model

Let the values are $\beta = 3, \gamma = 0.98, \delta = 10$, $\tilde{R} =$ **(R1 ,R2 ,R3 ,R4)=(2345,2445,2555,2655)**

$\tilde{L} =$ **(L1 ,L2 ,L3 ,L4)=(90,95,105,110),**$\tilde{H} =$ **(H1,H2,H3,H4)= (4.4,4.5,5.4,5.8)**

$\widetilde{M}^* =$ **714.2857142857**

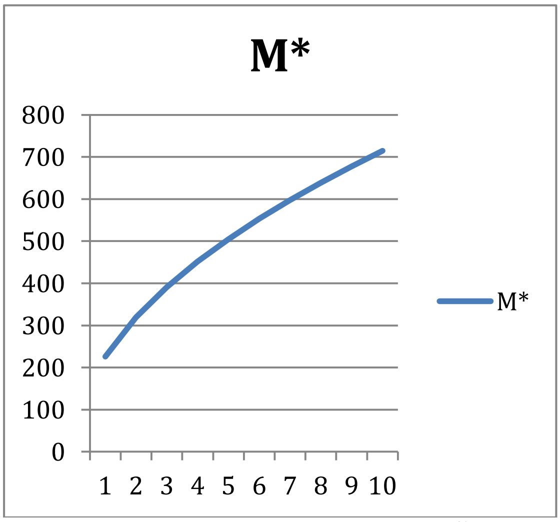
The minimization of fuzzy total production inventory cost is J$\tilde{T}$C(M)* = 365.

S.no	δ	$\widetilde{M}^*$	J$\tilde{T}$C(M)*
1	1	225.8769757263	5286.4056378821
2	2	319.4382825000	2576.0643118126
3	3	391.2303982180	1647.9871327697
4	4	451.7539514526	1183.2517618382
5	5	505.0762722761	906.03030380330
6	6	553.2833351725	722.84393343290
7	7	597.6143046672	593.33712769480
8	8	638.8765649999	497.27411692980
9	9	677.6309271789	423.40845145970
10	**10**	**714.2857142857**	**365.00000000000**

From the above table we observed that:

- **(i)** **The** $\widetilde{M}^*$ **Values are obtained by GM Integration is equal to crisp** $\widetilde{M}^*$ **values**
- **(ii)** **J$\tilde{T}$C(M)* obtained by GM Integration is equal to crisp J$\tilde{T}$C(M)***

M^* **value for various** δ **values** $\tilde{J}$C(M)* **value for various** δ **values**



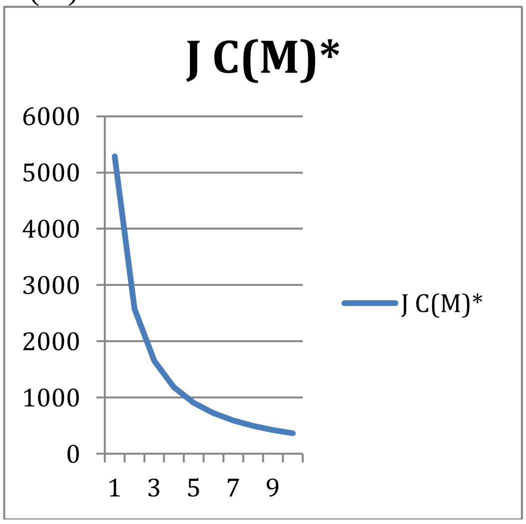


Figure 5.1 $\tilde{T}$C (M)* and M* value for various δ values

6. CONCLUSION

In this paper presents fuzzy models for an optimal integrated inventory model and minimizing the total cost. During this model M is stated as a fuzzy number and also the above total cost we replace into the fuzzy parameters. For every fuzzy model, a technique of defuzzification using under GM integration representation is applied to search out the estimate of total cost. Then illustrates the total cost on numerical example.

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