A RETAILER TOTAL PROFIT OF AN EOQ INVENTORY UNDER TRADE CREDIT

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ABSTRACT:

This paper concurrence with an economic order quantity (EOQ) inventory models under both non-linear stock and holding cost. A retailer's point is formed this inventory model. This EOQ inventory model is optimized using Lagrange method. The fuzzification and defuzzification has been done by pentagonal numbers and signed distance method. A some mathematical model is utilized to discover the helpfulness of the proposed combination models. At last, the affectability of the arrangement has been examined with the adjustments in the estimations of various boundaries related with the model.

KEYWORDS: Lagrange Method, Fuzzification, Defuzzification, Pentagonal Numbers and Signed Distance Method.

1. INTRODUCTION:

Throughout the most recent couple of years, much consideration has been pulled in to inventory systems, since the inventory models deal with the degree of inventories in the organizations proficiently and viably. In 1913[1,2],the principal stock model was presented by Harris. He built up the notable economic order quantity (EOQ) inventory model which thinks about that the demand is steady and known. Harris is considered as the establishing father of the inventory hypothesis. In 1934 ,Wilson[3] stimulated interest in the EOQ model in scholastics and businesses. Afterward, In 1963, Hadley, G., Whitin T.M., [4] examined many inventory systems.

Since Zadeh [5] establishing the fuzzy number concept and Chang and Zadeh [6]proposed the fuzzy mapping documentation, these points have been broadly concentrated by numerous creators. The fuzzy optimization has been one of the exploration lines. From a reasonable perspective, it is normally hard to manufacture a crisp model since all in all, the genuine issues have inalienable uncertainty and/or inaccuracy .Since it is the typical state, we consider fuzzy Lagrange method for optimization techineque.

Mathematical programming hypothesis and strategies are significant segments of optimization. There are a ton of deals with optimality conditions for fuzzy problems, which are a proof of the incredible interest that this point excites among specialists. The differentiability and convexity ideas are fundamental to portray the arrangements set for an optimization issue utilizing optimality conditions.

R. Kalaiarasi ,M.Sumathi and M.Sabina Begum[7-10] was developed in this method,The fuzzification and defuzzification has been done by pendagonal numbers and Signed distance method In the first place, we examine about fuzzy integrated inventory model with various circumstance. second, we enclose with numerical example.

2. SIGNED DISTANCE METHOD

Signed Distance Method For pentagonal fuzzy number is

$$P\,(\widetilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + 2a_4 + a_5}{8}$$

3. MATHEMATICAL MODEL

Notations: X purchasing cost, Y selling price ,A_b shortage cost,ν The elasticity of holding cost, E_0 replenishment cost, θ demand rate , D Inventory level at any time t ,TP (D) the total profit per unit time

3.1 MATHEMATICAL MODEL:

The integrated inventory model is

$$TP(D) = \frac{(XN - A_b)H + \theta XH}{D} + D(2\theta Y - E_0\ \theta)$$

Differentiating partially equation with respect to D,

$$\frac{\partial TP(D)}{\partial D} = -\left(\frac{(XN - A_b)H + \theta XH}{D^2}\right) + 2\theta Y - E_0\,\theta$$

Put $\frac{\partial TP(D)}{\partial D} = 0$ We get $D = \sqrt{\left(\frac{(XN - A_b)H + \theta XH}{2\theta Y - E_0\,\theta}\right)}$

4.1 The proposed inventory model for crisp quantity

We introduce an integrated inventory model TP (D) is

$$TP(D) = \frac{(XN - A_b)H_1 + \theta XH_1}{D} + D(2\theta Y_1 - E_0\ \theta)$$

Suppose, $\widetilde{H}$ = (H_1, H_2, H_3, H_4, H_5), $\widetilde{Y}$ = (Y_1, Y_2, Y_3, Y_4, Y_5)

$$TP(D) = \left[\left[\frac{(XN - A_b)H_1 + \theta XH_1}{D} + D(2\theta Y_1 - E_0\ \theta)\right], \left[\frac{(XN - A_b)H_2 + \theta XH_2}{D} + D(2\theta Y_2 - E_0\ \theta)\right], \left[\frac{(XN - A_b)H_3 + \theta XH_3}{D} + D(2\theta Y_3 - E_0\ \theta)\right], \left[\frac{(XN - A_b)H_4 + \theta XH_4}{D} + D(2\theta Y_4 - E_0\ \theta)\right], \left[\frac{(XN - A_b)H_5 + \theta XH_5}{D} + D(2\theta Y_5 - E_0\ \theta)\right]\right]$$

$$TP(D) = \frac{1}{8}\left[\left[\frac{(XN - A_b)H_1 + \theta XH_1}{D} + D(2\theta Y_1 - E_0\ \theta)\right] + 2\left[\frac{(XN - A_b)H_2 + \theta XH_2}{D} + D(2\theta Y_2 - E_0\ \theta)\right] + 2\left[\frac{(XN - A_b)H_3 + \theta XH_3}{D} + D(2\theta Y_3 - E_0\ \theta)\right] + 2\left[\frac{(XN - A_b)H_4 + \theta XH_4}{D} + D(2\theta Y_4 - E_0\ \theta)\right] + \left[\frac{(XN - A_b)H_5 + \theta XH_5}{D} + D(2\theta Y_5 - E_0\ \theta)\right]\right]$$

$$\frac{\partial TP(D)}{\partial D} = \frac{1}{8}\left[\left[-\left(\frac{(XN - A_b)H_1 + \theta XH_1}{D^2}\right) + 2\theta Y_1 - E_O\theta\right] + 2\left[-\left(\frac{(XN - A_b)H_2 + \theta XH_2}{D^2}\right) + 2\theta Y_2 - E_O\theta\right] + 2\left[-\left(\frac{(XN - A_b)H_3 + \theta XH_3}{D^2}\right) + 2\theta Y_3 - E_O\theta\right] + 2\left[-\left(\frac{(XN - A_b)H_4 + \theta XH_4}{D^2}\right) + 2\theta Y_4 - E_O\theta\right] + \left[-\left(\frac{(XN - A_b)H_5 + \theta XH_5}{D^2}\right) + 2\theta Y_5 - E_O\theta\right]\right]$$

Let us, take a partially differentiation is equal to 0

$$D^* = \sqrt{\frac{(XN - A_b)(H_1 + 2H_2 + 2H_3 + 2H_4 + H_5) + \theta H(H_1 + 2H_2 + 2H_3 + 2H_4 + H_5)}{2(\theta Y_5 + \theta Y_4 + \theta Y_3 + \theta Y_2 + \theta Y_1) - 8E_O\theta}}$$

4.2 A proposed Inventory Model for fuzzy quantity

Here, we can get the fuzzy total production inventory cost

P$[TP(D)] =$

$\frac{1}{8}\left[\left[\frac{(XN-A_b)H_1+\theta XH_1}{D} + D(2\theta Y_1 - E_0\ \theta)\right], 2\left[\frac{(XN-A_b)H_2+\theta XH_2}{D} + D(2\theta Y_2 - E_0\ \theta)\right], 2\left[\frac{(XN-A_b)H_3+\theta XH_3}{D} +\right.\right.$

D2θY3−E0 θ,2XN−Ab H4+θXH4D+D2θY4−E0 θ,XN−Ab H5+θXH5D+D2θY5−E0 θ

$$P[TP(D)] = \frac{1}{8}\left[\left[\frac{(XN - A_b)H_1 + \theta XH_1}{D} + D(2\theta Y_1 - E_0\ \theta)\right] + 2\left[\frac{(XN - A_b)H_2 + \theta XH_2}{D} + D(2\theta Y_2 - E_0\ \theta)\right] + 2\left[\frac{(XN - A_b)H_3 + \theta XH_3}{D} + D(2\theta Y_3 - E_0\ \theta)\right] + 2\left[\frac{(XN - A_b)H_4 + \theta XH_4}{D} + D(2\theta Y_4 - E_0\ \theta)\right] + \left[\frac{(XN - A_b)H_5 + \theta XH_5}{D} + D(2\theta Y_5 - E_0\ \theta)\right]\right] \quad (*)$$

With 0 < D1 ≤ D2 ≤ D3 ≤ D4≤ D_5**.It will not change the (*) if we replace inequality conditions 0<D1 ≤ D2 ≤ D3 ≤ D4≤** D_5 **into the following inequality D2 - D1 ≥ 0, D3 – D2 ≥ 0, D4 - D3 ≥ 0,** $D_5 - D_4$**,D1 > 0.**

We use Lagrange a method to find minimize TP (D) in (3)

Case 1: To find the min P[TP(D)], $\frac{\partial T}{\partial D_1} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_1 + \theta XH_1}{D^2}\right) + 2\theta Y_1 - E_O\theta\right]$

$\frac{\partial T}{\partial D_2} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_2 + \theta XH_2}{D^2}\right) + 2\theta Y_2 - E_O\theta\right]$, $\frac{\partial T}{\partial D_3} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_3 + \theta XH_3}{D^2}\right) + 2\theta Y_3 - E_O\theta\right]$ $\frac{\partial T}{\partial D_4} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_4 + \theta XH_4}{D^2}\right) + 2\theta Y_4 - E_O\theta\right]$ $\frac{\partial T}{\partial D_5} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_5 + \theta XH_5}{D^2}\right) + 2\theta Y_5 - E_O\theta\right]$

Let $\frac{\partial T}{\partial D_1} = 0, D_1 = \sqrt{\left(\frac{(XN-A_b)H_1+\theta XH_1}{2\theta Y_5 - E_0\ \theta}\right)}$**,Let** $\frac{\partial T}{\partial D_2} = 0, D_2 = \sqrt{2\left(\frac{(XN-A_b)H_2+\theta XH_2}{2\theta Y_4 - E_0\ \theta}\right)}$

Let $\frac{\partial T}{\partial D_3} = 0, D_3 = \sqrt{2\left(\frac{(XN-A_b)H_3+\theta XH_3}{2\theta Y_3 - E_0\ \theta}\right)}$**,Let** $\frac{\partial T}{\partial D_4} = 0, D_4 = \sqrt{2\left(\frac{(XN-A_b)H_4+\theta XH_4}{2\theta Y_2 - E_0\ \theta}\right)}$

Let $\frac{\partial T}{\partial D_5} = 0, D_5 = \sqrt{\left(\frac{(XN-A_b)H_5+\theta XH_5}{2\theta Y_1 - E_0\ \theta}\right)}$

Because the above see that D1 > D2 > D3 > D4>D5. It is not satisfying the constraint 0 < D1 > D2 > D3 > D4 >D5. Put G = 1 so go case 2

Case 2: Convert the inequality constraint D2 – D1 ≥ 0 into equality constraint D2 – D1 = 0 and We have Lagrangean function as L (D1, D2, D3, D4,D5, λ**) =P[TP(D)]-** λ**(D2 – D1)**

$$\frac{\partial L}{\partial D_1} = \left[-\left(\frac{(XN - A_b)H_1\ + \theta XH_1}{D^2}\right) + 2\theta Y_1\ - E_O\theta\right]\frac{1}{8} - \lambda - \left[-\left(\frac{(XN - A_b)H_1\ + \theta XH_1}{D^2}\right) + 2\theta Y_1\ - E_O\theta\right] = 8\lambda$$

$$\frac{\partial L}{\partial D_2} = \frac{1}{8}\left[-\left(\frac{(XN - A_b)H_2\ + \theta XH_2}{D^2}\right) + 2\theta Y_2\ - E_O\theta\right] - \lambda = 0$$

$$\frac{\partial L}{\partial D_3} = \frac{1}{8}\left[-\left(\frac{(XN - A_b)H_3\ + \theta XH_3}{D^2}\right) + 2\theta Y_3\ - E_O\theta\right] = 0$$

$$\frac{\partial L}{\partial D_4} = \frac{1}{8}\left[-\left(\frac{(XN - A_b)H_4\ + \theta XH_4}{D^2}\right) + 2\theta Y_4\ - E_O\theta\right] = 0$$

$$\frac{\partial L}{\partial D_5} = \frac{1}{8}\left[-\left(\frac{(XN - A_b)H_5\ + \theta XH_5}{D^2}\right) + 2\theta Y_5\ - E_O\theta\right] = 0$$

$$\frac{\partial T}{\partial \lambda} = -(D_2 - D_1)$$

$D_1 = D_2 = \sqrt{\frac{(XN-A_b)(H_1+2H_2)+\theta X(H_1+2H_2)}{2(\theta Y_5+2\theta Y_4)-3E_0\theta}}$, $D_3 = \sqrt{2\left(\frac{(XN-A_b)H_3+\theta XH_3}{2\theta Y_3-E_0\theta}\right)}$

$D_4 = \sqrt{2\left(\frac{(XN-A_b)H_4+\theta XH_4}{2\theta Y_2-E_0\theta}\right)}$, $D_5 = \sqrt{\left(\frac{(XN-A_b)H_5+\theta XH_5}{2\theta Y_1-E_0\theta}\right)}$

Because the above results show that D3 > D4, it does not satisfy the constraint 0 < D1 > D2 > D3 > D4>D5. Put G = 2 so go to case 3.

Case 3: Turn the inequality constraints D2 – D1 ≥ 0, into equality constraints D2 – D1 = 0 and D3 – D1 = 0. We optimize P [TP (D)]. Then the Lagrangean method is

L (D1, D2, D3, D4, D5, λ1,λ2) = P [TP (D)] – λ1(D2 – D1) – λ2(D3 – D2)

$$\frac{\partial L}{\partial D_1} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_1 + \theta XH_1}{D^2}\right) + 2\theta Y_1 - E_O\theta\right] + \lambda_1 = 0$$
$$\frac{\partial L}{\partial D_2} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_2 + \theta XH_2}{D^2}\right) + 2\theta Y_2 - E_O\theta\right] + \lambda_2 - \lambda_1 = 0$$
$$\frac{\partial L}{\partial D_3} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_3 + \theta XH_3}{D^2}\right) + 2\theta Y_3 - E_O\theta\right] - \lambda_2 = 0$$
$$\frac{\partial L}{\partial D_4} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_4 + \theta XH_4}{D^2}\right) + 2\theta Y_4 - E_O\theta\right] = 0$$
$$\frac{\partial L}{\partial D_5} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_5 + \theta XH_5}{D^2}\right) + 2\theta Y_5 - E_O\theta\right] = 0$$
$$\frac{\partial L}{\partial \lambda_1} = -(D_2 - D_1), \frac{\partial L}{\partial \lambda_2} = -(D_3 - D_2)$$
$$D_1 = D_2 = D_3 = \sqrt{\frac{(XN-A_b)(H_1+2H_2+2H_3)+\theta X(H_1+2H_2+2H_3)}{2(\theta Y_5+2\theta Y_4+2\theta Y_3)-5E_0\theta}}$$
$$D_4 = \sqrt{2\left(\frac{(XN-A_b)H_4+\theta XH_4}{2\theta Y_2-E_0\theta}\right)}, D_5 = \sqrt{\left(\frac{(XN-A_b)H_5+\theta XH_5}{2\theta Y_1-E_0\theta}\right)}$$

The above results D1 > D4, does not satisfy the constraint 0 < D1 > D2 > D3 > D4> D5.
Put G = 3 so go to case 4.

Case 4: Turn the inequality constraints D2 – D1 ≥ 0, D3 – D2 ≥ 0 and D4 – D3 ≥ 0 into equality constraints D2 – D1 = 0, D3 – D1 = 0 and D4 – D3 = 0.

The Lagrangean function is given by

L(D1, D2, D3, D4, D5, λ1,λ2,λ3) = P[TP(D)]- λ1(D2 – D1) – λ2(D3 – D2) – λ3(D4 – D3)

$$\frac{\partial L}{\partial D_1} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_1 + \theta XH_1}{D^2}\right) + 2\theta Y_1 - E_O\theta\right] + \lambda_1 = 0$$
$$\frac{\partial L}{\partial D_2} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_2 + \theta XH_2}{D^2}\right) + 2\theta Y_2 - E_O\theta\right] - \lambda_1 + \lambda_2 = 0$$
$$\frac{\partial L}{\partial D_3} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_3 + \theta XH_3}{D^2}\right) + 2\theta Y_3 - E_O\theta\right] - \lambda_2 = 0$$
$$\frac{\partial L}{\partial D_4} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_4 + \theta XH_4}{D^2}\right) + 2\theta Y_4 - E_O\theta\right] = 0$$
$$\frac{\partial L}{\partial D_5} = \frac{1}{8}\left[-\left(\frac{(XN-A_b)H_5 + \theta XH_5}{D^2}\right) + 2\theta Y_5 - E_O\theta\right] = 0$$
$$\frac{\partial L}{\partial \lambda_1} = -(D_2 - D_1), \frac{\partial L}{\partial \lambda_2} = -(D_3 - D_2), \frac{\partial L}{\partial \lambda_3} = -(D_4 - D_3)$$
$$D_1 = D_2 = D_3 = D_4 = \sqrt{\frac{(XN-A_b)(H_1+2H_2+2H_3+2H_4)+\theta X(H_1+2H_2+2H_3+2H_4)}{2(\theta Y_5+2\theta Y_4+2\theta Y_3+2\theta Y_2)-7E_0\theta}}$$

The above results D1 > D5, does not satisfy the constraint 0 < D1 > D2 > D3 > D4> D5.
Put G = 4so go to case 5.

Case 5: Turn the inequality constraints D2 – D1 ≥ 0, D3 – D2 ≥ 0 ,D4 – D3 ≥ 0 D5 – D4 ≥ 0 into equality constraints D2 – D1 = 0, D3 – D1 = 0 and D4 – D3 = 0, D5 – D4 = 0

The Lagrangean function is given by

L(D1, D2, D3, D4, D5, λ1,λ2,λ3,λ_4) = P[TP(D)]- λ1(D2 – D1) – λ2(D3 – D2) – λ3(D4 – D3)-$\lambda_4(D_5 - D_4)$

$$\frac{\partial L}{\partial D_1} = \frac{1}{8}\left[-\left(\frac{(XN - A_b)H_1 \ + \theta X H_1}{D^2}\right) + 2\theta Y_1 \ - E_O\theta\right] + \lambda_1 = 0$$

$$\frac{\partial L}{\partial D_2} = \frac{1}{8}\left[-\left(\frac{(XN - A_b)H_2 \ + \theta X H_2}{D^2}\right) + 2\theta Y_2 \ - E_O\theta\right] - \lambda_1 + \lambda_2 = 0$$

$$\frac{\partial L}{\partial D_3} = \frac{1}{8}\left[-\left(\frac{(XN - A_b)H_3 + \theta X H_3}{D^2}\right) + 2\theta Y_3 \ - E_O\theta\right] - \lambda_2 = 0$$

$$\frac{\partial L}{\partial D_4} = \frac{1}{8}\left[-\left(\frac{(XN - A_b)H_4 \ + \theta X H_4}{D^2}\right) + 2\theta Y_4 \ - E_O\theta\right] - \lambda_3 = 0$$

$$\frac{\partial L}{\partial D_5} = \frac{1}{8}\left[-\left(\frac{(XN - A_b)H_5 \ + \theta X H_5}{D^2}\right) + 2\theta Y_5 \ - E_O\theta\right] = 0$$

$$\frac{\partial L}{\partial \lambda_1} = -(D_2 - D_1), \frac{\partial L}{\partial \lambda_2} = -(D_3 - D_2), \frac{\partial L}{\partial \lambda_3} = -(D_4 - D_3), \frac{\partial L}{\partial \lambda_4} = -(D_5 - D_4)$$

$$D^* = D_1 = D_2 = D_3 = D_4 = D_5$$

$$= \sqrt{\frac{(XN - A_b)(H_1 + 2H_2 + 2H_3 + 2H_4 + H_5) + \theta X(H_1 + 2H_2 \ + 2H_3 + 2H_4 + H_5)}{2(\theta Y_5 + 2\theta Y_4 + 2\theta Y_3 + 2\theta Y_2 + \theta Y_1) - 8E_0\theta}}$$

5. NUMERICAL EXAMPLES

Crisp model

The input parameters are H=2.5,X=2,Y=6,A_b=0.5,E_0=7 ,N=3 ,$\theta = 4$. We get D **= 1.2990, TP (D)=45.6105**

Fuzzy model

The input parameters are $\widetilde{H}$ = (H1, H2, H3, H4, H5) = (0.5, 1.5, 2.5, 3.5, and 4.5)

$\widetilde{Y}$ = (Y1, Y2, Y3, Y4, Y5) = (4, 5, 6, 7, 8)

Y=6 ,θ=4 ,A_b=0.5 ,E_0=0 ,X=2 ,N=3.

We fine the optimal fuzzy production quantity,

D^* = 1.2990

The minimization fuzzy total production inventory cost is

TP (D)=(45.6105)

S,NO	N	D^*	TP (D)
1	1	1.0872	42.5312
2	2	1.1989	44.0708
3	3	1.299	45.6105
4	4	1.3919	47.1501
5	5	1.479	48.6898
6	6	1.5612	50.2294
7	7	1.6394	51.7691

D^* **value for various values** **TP (D) value for various values**

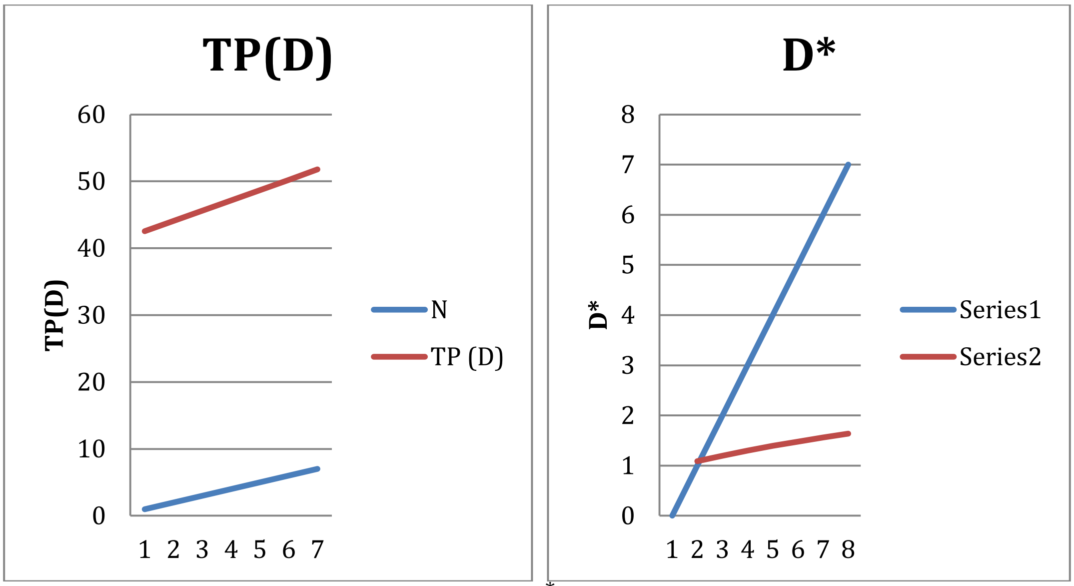

Figure 5.1 TP (D) and D^* **value for various** N **values**

6. CONCLUSION:

For an efficient integrated inventory model, this paper introduce two fuzzy models and minimizing the overall expected cost of both non – linear stock and holding cost.In first model the inventory parameters are taken as crisp . Secondly, changed crisp to fuzzy using pentagonal fuzzy numbers, next ,changed fuzzy to crisp using signed distance method.the optimization is done by non linear programming method such as lagrange method.Finally we get the maximize the optimal order quantity and minimize the optimal total cost for an integrated inventory model.

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